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A UNIVERSAL ALGORITHM FOR THE SOLUTION OF PROBLEMS INVOLVING THE MATHEMATICAL MODELING OF THE THERMAL REGIME IN A STRUCTURE, IN ONE-DIMENSIONAL APPROXIMATION

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We examine a universal solution algorithm for problems related to the mathematical modeling of the heat regime in structures in one-dimensional approximation, synthesizing the possibilities and advantages of the solution algorithms of these problems, as determined from graphs of general form and a graph in the form of a tree.

The method of mathematical modeling of the thermal regime in structures in one-dimensional approximation [1-3] has recently found widespread application with regard to problems of thermal designs in various heat-engineering systems and devices. The thermal model of a structure in this case is represented in the form of a graph, on N of whose arms are given the equations of heat conduction modeling the thermal state in distributed structural elements, with the heat-balance equation for concentrated elements in combination with conditions of thermal stress given at the N_V apices (at $N_{\alpha_{in}}$ internal apices of the graph) or by the boundary conditions (at N_{α_b} boundary apices). We will identify the boundary apices of the graph as those apices with which only a single arm is associated. The inside apices will include all those with which a minimum of two arms are connected.

As a rule, the system of nonsteady nonuniform one-dimensional heat-conduction equations with which we are dealing here, as a result of the finite-difference approximation of the differential operators, reduces to a system of algebraic equations determined on the graph of the thermal model for the solution of which various modifications of the parametric sweeping method [2] is used, or where use is made of a generalized algorithm [3], utilizing a cyclical sweeping method. These methods exhibit excellent convergence and stability and are suitable for thermal models whose graphs are arbitrary in form (see Fig. 1a), i.e., it contains cycles, loops, etc. The application of these methods requires a considerable number of arithmetic operations and, consequently, considerable computer capacity.

The original graph can frequently be represented as some combination (total) of simpler interconnected graphs. This makes it possible to break down the graph in the following manner. Let us assume that N_b boundary graphs are contained within the original graph; these boundary graphs simulate the characteristics of a tree (or bush), and there is also a root graph of general form (i.e., with loops and cycles) to link all of the separate graphs into a graph representing the thermal model of the structure. In this case, the root apices of the boundary graphs are the inside apices of the root graph. If we were to include the simplest tree-shaped graphs consisting of a single arm and two terminal apices in the number of boundary graphs, all of the apices of the root graph would be inside graphs in terms of the earlier-introduced definition.

The α apices of the graph under consideration have been determined on the set $V = V_r + \sum_{p=1}^{N_r} V_p$. The total number of N_V apices in the original graph consists of the N_{α_r} apices

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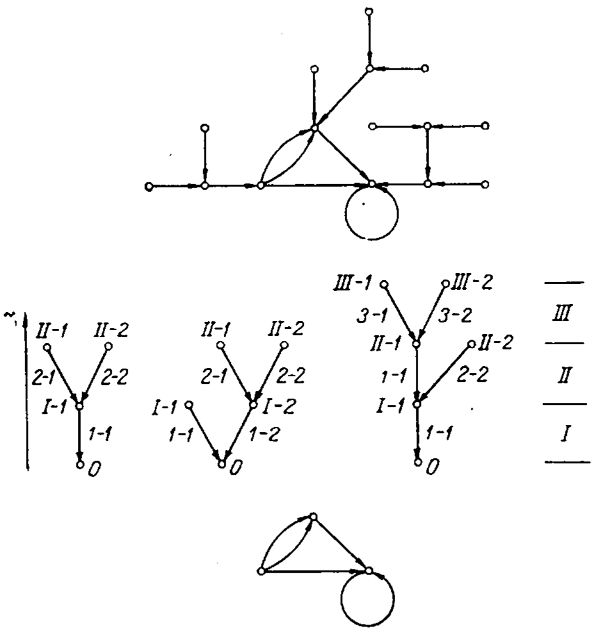

a

b

c

Fig. 1. Orientational graph of a thermal model and its decomposition: a) graph of the thermal model; b) boundary graphs of the model) c) root graph of the model.

of the root graph and $\sum_{p=1}^{N_b} (N_{\alpha_p} - 1)$ apices of the boundary graphs, i.e., $N_V = N_{\alpha r} + \sum_{p=1}^{N_b} (N_{\alpha_p} - 1)$. The arms of the graph form a three-dimensional region $D = D_r + \sum_{p=1}^{N_b} D_p$, the number N arms of the graph is composed of the number of arms N_r of the root graph and the number of arms $\sum_{p=1}^{N_b} N_p$ of the boundary graphs $\left(N = N_r + \sum_{p=1}^{N_b} N_p\right)$.

Let us assume that the structure, orientation, and the numbering of the apices and arms of the root graph is arbitrary. Each pair of apices α and β of the root graph can be combined with several arms $E_{\alpha\beta}{}^k = (x_\alpha, x_\beta)k$, $k = 1, 2, \ldots, M_{\alpha\beta} = (\mathrm{const})_{\alpha\beta}$ (in which case $M_{\alpha\beta} = M_{\beta\alpha}$). Moreover, the root graph may contain loops $E_{\alpha\alpha}{}^k = (x_\alpha, x_\alpha)k$, $k = 1, 2, \ldots,$ $M_{\alpha\alpha} = (\mathrm{const})_{\alpha\alpha}$. Let us establish the apex $\alpha \in V_r$ and let us examine the β apices such that the arms $E_{\alpha\beta}$ (emanating from the apex α) and $E_{\beta\alpha}$ (incident at apex α) belong to V_r. We will assume that $\beta \in G_\alpha{}^-$ (or $\beta \in G_\alpha{}^+$), if $E_{\alpha\beta} \in V_r$ (or $E_{\beta\alpha} \in V_r$), $G_\alpha = G_\alpha{}^+ + G_\alpha{}^-$. Correspondingly, the number of arms in the initial number of apices under consideration is determined by the relationship $N_\alpha = N_\alpha{}^+ + N_\alpha{}^-$.

For the sake of convenience in organizing the calculation process, let us introduce the following construction and numbering system for the apices and arms of the boundary graphs in terms of levels, which are determined in the following way. We will assume the root apex of the boundary graph to be the zero level. The arm (or arms, if the graph under consideration is in the form of a bush), which is connected to the root apex, will be referred to as the arm (arms) of the first level. If the number of arms is greater than unity, we then propose to number them from left to right. We will treat the first-level apices as the initial apices of the first-level arms (see the orientation of the arms in Fig. 1b). The construction of the apices and the numbering of the arms for the subsequent levels are carried out in the same way. When the graph is oriented in the manner shown in Fig. 1b, the root apex of the boundary graph is the terminal.

In accordance with the structure of the boundary graph, the set of its apices $V_p = \sum_{\gamma=0}^{N_y^p} V_\gamma^p$. Here it is assumed that the root apex pertains to the zero level, i.e., $V_0P = 1$.

Taking into consideration that the apices of the boundary graph may be both inside and boundary, we will assume that the set $V_\gamma P$ at each level consists of the sum of the sets of the inside apices $V_{\gamma in}P$ and the boundary apices $V_{\gamma_b}P$, i.e., $V_\gamma P = V_{\gamma in}P + V_{\gamma_b}P$, $\forall \gamma \in \{1, N_\gamma^p\}$.

Keeping in mind the above assumptions and definitions, let us examine the mathematical model of the thermal regime of a structure in one-dimensional approximation:

$$\forall j \in \{1, 2, \ldots, N_r\} \wedge \forall j \in \{1, 2, \ldots, N_p\},\ p = \overline{1, N_b}:$$

$$\rho(x_j, T) C_p(x_j, T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x_j} \left(\lambda(x_j, T) S(x_j) \frac{\partial T}{\partial x_j} \right) \frac{1}{S(x_j)} + q_V(x_j, T, t); \tag{1}$$

$$T(x_j, t)|_{t=0} = T_0(x_j); \tag{2}$$

$$\forall \alpha \in V_r,\quad p = \overline{1, N_b}:$$

$$V_\alpha \rho_\alpha(T) C_{p_\alpha}(T) \frac{dT(x_\alpha)}{dt} = \sum_{j=1}^{N_\alpha^+} -\lambda(x_j, T) S(x_j) \frac{\partial T}{\partial x_j}\Big|_{x_j=L_j} + \sum_{j=1}^{N_1^-} \lambda(x_j, T) S(x_j) \frac{\partial T}{\partial x_j}\Big|_{x_j=0} + Q_{V_\alpha}(T, t) + (1 - \gamma_{\alpha,p}) \sum_{l=1}^{N_\alpha^{p+}} -\lambda(x_l, T) S(x_l) \frac{\partial T}{\partial x_l}\Big|_{x_l=L_l^p}, \tag{3}$$

where

$$\gamma_{\alpha,p} = \begin{cases} 1, & \text{if } \gamma \in V_r \setminus (V_r \cap V_p),\ p = \overline{1, N_b}; \\ 0, & \text{if } \gamma \in V_r \cap V_p,\ p = \overline{1, N_b}; \end{cases} \tag{4}$$

$$\forall \alpha \in V_p \setminus (V_r \cap V_p),\quad p = \overline{1, N_b}:$$

$$V_\alpha \rho_\alpha(T) C_{p_\alpha}(T) \frac{dT(x_\alpha)}{dt} = \sum_{j=1}^{N_\alpha^{p+}} -\lambda(x_j, T) S(x_j) \frac{\partial T}{\partial x_j}\Big|_{x_j=L_j} + \lambda(x_j, T) S(x_j) \frac{\partial T}{\partial x_j}\Big|_{x_j=0,\, x_j \in D_\alpha^p} + Q_{V_\alpha}(T, t); \tag{5}$$

$$\forall \alpha \in V \quad T(x_\alpha)|_{t=0} = T_0(x_\alpha). \tag{6}$$

Let us now turn to the construction of a solution algorithm for the problem under consideration. As a result of the finite-difference approximation of (1)-(6) on a six-point two-layer pattern based on an implicit scheme, we will obtain

$$\forall j \in \{1, 2, \ldots, N_r\}:$$

$$\left.\begin{aligned} &\hat{T}_{j,0}(E_{\alpha\beta}^k) \equiv \hat{T}(x_\alpha); \\ &\forall i = \overline{1, N_j - 1}: \\ &A_{j,i}(E_{\alpha\beta}^k) \hat{T}_{j,i-1}(E_{\alpha\beta}^k) - C_{j,i}(E_{\alpha\beta}^k) \hat{T}_{j,i}(E_{\alpha\beta}^k) + \\ &+ B_{j,i}(E_{\alpha\beta}^k) \hat{T}_{j,i+1}(E_{\alpha\beta}^k) = -F_{j,i}(E_{\alpha\beta}^k); \\ &\hat{T}_{j,N_j}(E_{\alpha\beta}^k) \equiv \hat{T}(x_\beta); \end{aligned}\right\} \tag{7}$$

$$\forall \alpha \in V_r \wedge p \in \{1, 2, \ldots, N_b\}:$$

$$\frac{1}{h_t} V_\alpha \hat{\rho}_\alpha \hat{C}_{p_\alpha} (\hat{T}(x_\alpha) - T(x_\alpha)) = \sum_{j=1}^{N_\alpha^+} \sum_{k=1}^{M_{\beta\alpha}} \hat{\lambda}_{j,N_j} (E_{\beta\alpha}^k) S_{j,N_j} (E_{\beta\alpha}^k) \times$$

$$\times [A_{j,N_j}(E_{\beta\alpha}^k) \hat{T}_{j,N_j}(E_{\beta\alpha}^k) - C_{j,N_j}(E_{\beta\alpha}^k) \hat{T} + F_{j,N_j}(E_{\beta\alpha}^k)] +$$

$$+ \sum_{j=1}^{N_\alpha^-} \sum_{k=1}^{M_{\alpha\beta}} \hat{\lambda}_{j,0} (E_{\alpha\beta}^k) S_{j,0} (E_{\alpha\beta}^k) [- C_{j,0} (E_{\alpha\beta}^k) \hat{T}(x_\alpha) +$$

$$+ B_{j,0}(E_{\alpha\beta}^k) \hat{T}_{j,1}(E_{\alpha\beta}^k) + F_{j,0}(E_{\alpha\beta}^k)] + \hat{Q}_{V_\alpha} +$$

$$+ (1 - \gamma_{\alpha,p}) \sum_{l=1}^{N_1^{p+}} \hat{\lambda}_{l,N_l^p} S_{l,N_l^p} (A_{l,N_l^p} \hat{T}_{l,N_l^p-1} - C_{l,N_l^p} \hat{T}(x_\alpha) + F_{l,N_l^p}); \tag{8}$$

$$\left.\begin{array}{c} \forall j \in \{1, 2, \ldots, N_p\},\ p = \overline{1, N_b}: \\ \hat{T}_{j,0} \equiv \hat{T}_\alpha; \\ \forall i = \overline{1, N_j^p - 1}: \\ A_{j,i}\hat{T}_{j,i-1} - C_{j,i}\hat{T}_{j,i} + B_{j,i}\hat{T}_{j,i+1} = -F_{j,i}; \\ \hat{T}_{j,N_j^p} \equiv \hat{T}_\beta; \end{array}\right\} \tag{9}$$

$$\forall \alpha \in V_p \setminus (V_p \cap V_r),\quad p = \overline{1, N_b}:$$

$$\frac{1}{h_t} V_\alpha \hat{\rho}_\alpha \hat{C}_{p_\alpha} (\hat{T}_\alpha - T_\alpha) = \sum_{j=1}^{N_\alpha^{p+}} \hat{\lambda}_{j,N_j} S_{j,N_j} (A_{j,N_j} \hat{T}_{j,N_j-1} - C_{j,N_j} \hat{T}_\alpha + F_{j,N_j}) +$$

$$+ \hat{\lambda}_{j,0} S_{j,0} (-C_{j,0} \hat{T}_\alpha + B_{j,0} \hat{T}_{j,1} + F_{j,0}) + \hat{Q}_{V_\alpha}; \tag{10}$$

$$\forall \alpha_p = 0,\quad p = \overline{1, N_b}:$$

$$\sum_{j=1}^{N_1^{p+}} \hat{\lambda}_{j,N_j^p} S_{j,N_j^p} (A_{j,N_j^p} \hat{T}_{j,N_j^p-1} - C_{j,N_j^p} \hat{T}_\alpha + F_{j,N_j^p}) = \Phi_{\alpha p}, \tag{11}$$

where $\Phi_{\alpha p}$ is the function modeling the thermal effect of the root graph on the p-th boundary graph at the point of their conjugacy [see (3), (8)].

Let us assume that for each p-th boundary graph the solution of system (9)-(11) can be presented in the form of ordinary sweeping:

$$\hat{T}_{j,i} = \alpha_{j,i+1} \hat{T}_{j,i+1} + \beta_{j,i+1},\quad i = \overline{1, N_j^p - 1}, \tag{12}$$

where

$$\alpha_{j,i+1} = \frac{B_{j,i}}{C_{j,i} - \alpha_{j,i} A_{j,i}}; \tag{13}$$

$$\beta_{j,i+1} = \frac{A_{j,i} B_{j,i} + F_{j,i}}{C_{j,i} - \alpha_{j,i} A_{j,i}}. \tag{14}$$

The boundary values of the sweeping coefficients at each boundary arm belonging to the upper level of the graph under consideration is defined, jointly considering (10) and (12), for i = 0. As a result, we obtain:

$$\alpha_{j,1} = \frac{M_\alpha}{L_\alpha}; \tag{15}$$

$$\beta_{j,1} = \frac{N_\alpha}{L_\alpha}, \tag{16}$$

where

$$L_\alpha = \frac{1}{h_t} V_\alpha \hat{\rho}_\alpha \hat{C}_{p_\alpha} + \hat{\lambda}_{j,0} S_{j,0} C_{j,i}; \tag{17}$$

$$M_\alpha = \hat{\lambda}_{j,0} S_{j,0} B_{j,0}; \tag{18}$$

$$N_\alpha = \frac{1}{h_t} V_\alpha \hat{\rho}_\alpha \hat{C}_{p_\alpha} T_\alpha + \hat{\lambda}_{j,0} S_{j,0} F_{j,0} + \hat{Q}_{V_\alpha}. \tag{19}$$

Knowing the boundary values of the sweeping coefficients, using (13) and (14), we find the value of the sweeping coefficients $\alpha_{j,i}$, $\beta_{j,i}$ $\forall i = \overline{1, N_j - 1}$. As a result, we eliminate all boundary apices of the upper level and turn to an examination of the apices in the following $N_y - 1$ level. At each γ-th intervening level of the boundary graph we have both the boundary apices $\alpha \in V_{\gamma_b}{}^p$, so that at least one inside apex $\alpha \in V_{\gamma_{in}}$. The principle of eliminating the boundary apices at the intervening level is analogous to the process of eliminating the boundary apices at the upper level of the graph under consideration that we have just considered. In eliminating the inside apices we examine jointly (10) and (12) for $i = N_j$, where $j \in N_\alpha{}^{p+}$, and for $i = 0$, where $j \in N_\alpha{}^{p-}$. The boundary values of the sweeping coefficients are then calculated in the following manner:

$$\alpha_{j,1} = \frac{M'_\alpha}{L'_\alpha}, \tag{20}$$

$$\beta_{j,1} = \frac{N'_\alpha}{L'_\alpha}, \tag{21}$$

where

$$L'_\alpha = \frac{1}{h_t}(1-\gamma_{\alpha,1}) V_\alpha \hat{\rho}_\alpha \hat{C}_{p_\alpha} - \sum_{j=1}^{N_\alpha^+} \hat{\lambda}_{j,N_j} S_{j,N_j} (A_{j,N_j}\alpha_{j,N_j} - C_{j,N_j}) + \hat{\lambda}_{j,0} S_{j,0} C_{j,0}; \tag{22}$$

$$M'_\alpha = \hat{\lambda}_{j,0} S_{j,0} B_{j,0}; \tag{23}$$

$$N'_\alpha = \frac{1}{h_t} V_\alpha \hat{\rho}_\alpha \hat{C}_{p_\alpha} T_\alpha + \sum_{j=1}^{N_\alpha^+} \hat{\lambda}_{j,N_j} S_{j,N_j} (A_{j,N_j}\beta_{j,N_j} + F_{j,N_j}) + \hat{\lambda}_{j,0} S_{j,0} F_{j,0} + \hat{Q}_{V_\alpha}. \tag{24}$$

As we make the transition from the upper to the lower level, we determine the sweeping coefficients in this manner at each arm of the graph. In this case, we have successive elimination of all boundary and inside apices of the graph, since the boundary conditions or conditions of thermal conjugacy given at these apices are used to calculate the boundary sweeping coefficients on the corresponding lower-level arms of the graph. The values of the sweeping coefficients on the root arm (arms) (and root in the case of a bush) of the graph are used to determine the thermal influence exerted by the p-th boundary graph on the root graph [see (11)]:

$$\hat{T}_\alpha \sum_{l=1}^{N_1^{p+}} \hat{\lambda}_{l,N_l^p} S_{l,N_l^p} (A_{l,N_l^p}\alpha_{l,N_l^p} - C_{l,N_l^p}) + \sum_{l=1}^{N_1^{p+}} (A_{l,N_l^p}\beta_{l,N_l^p} + F_{l,N_l^p}) = \Phi_{\alpha p}. \tag{25}$$

If we use this expression to describe the thermal state of the corresponding apex of the root graph, then we can examine the algorithm for the calculation of the temperature fields on the root graph (see [1]).

We will seek the solution of system (7), (8), with consideration of (25), at each arm of the root graph in the form:

$$\hat{T}_{j,i+1} = \alpha_{j,i}\hat{T}_{j,i} + \beta_{j,i} + \gamma_{j,i}\hat{T}(x_\beta) \text{ on } E^k_{\alpha\beta}\,(\beta \in G^-_\alpha); \tag{26}$$

$$\hat{T}_{j,i} = \bar{\alpha}_{j,i+1}\hat{T}_{j,i+1} + \bar{\beta}_{j,i+1} + \bar{\gamma}_{j,i+1}\hat{T}(x_\beta) \text{ on } E^k_{\beta\alpha}\,(\beta \in G^+_\alpha). \tag{27}$$

Using (26) for i = 0 and (27) for i = $N_j - 1$, it is easy to obtain the following system for the determination of the unknown temperature at the apices of the root graph:

$$\hat{T}(x_\alpha) = \sum_{\beta \in G_\alpha} A_{\alpha\beta}\hat{T}(x_\beta) + \Phi_\alpha, \quad \alpha = 1, 2, \ldots, N_\text{к}, \tag{28}$$

where

$$A_{\alpha\beta} = D^{-1}_\alpha \begin{cases} \sum\limits_{k=1}^{M_{\beta\alpha}} \hat{\lambda}_{j,N_j}(E^k_{\beta\alpha})\, S_{j,N_j}(E^k_{\beta\alpha})\, A_{j,N_j}(E^k_{\beta\alpha})\, \bar{\gamma}_{j,N_j}(E^k_{\beta\alpha}), & \beta \in G^+_\alpha; \\ \sum\limits_{k=1}^{M_{\alpha\beta}} \hat{\lambda}_{j,0}(E^k_{\alpha\beta})\, S_{j,0}(E^k_{\alpha\beta})\, B_{j,0}(E^k_{\alpha\beta})\, \gamma_{j,0}(E^k_{\alpha\beta}), & \beta \in G^-_\alpha; \end{cases} \tag{29}$$

$$\begin{aligned} \Phi_\alpha = D^{-1}_\alpha \Big\langle V_\alpha \hat{\rho}_\alpha \hat{C}_{p_\alpha} T(x_\alpha) + \sum_{j=1}^{N^+_\alpha} \sum_{k=1}^{M_{\beta\alpha}} \hat{\lambda}_{j,N_j}(E^k_{\beta\alpha})\, S_{j,N_j}(E^k_{\beta\alpha}) \times \\ \times [A_{j,N_j}(E^k_{\beta\alpha})\, \bar{\beta}_{j,N_j}(E^k_{\beta\alpha}) + F_{j,N_j}(E^k_{\beta\alpha})] + \sum_{j=1}^{N^-_\alpha} \sum_{k=1}^{M_{\alpha\beta}} \hat{\lambda}_{j,0}(E^k_{\alpha\beta})\, S_{j,0}(E^k_{\alpha\beta}) \times \\ \times [B_{j,0}(E^k_{\alpha\beta})\, \beta_{j,0}(E^k_{\alpha\beta}) + F_{j,0}(E^k_{\alpha\beta})] + \\ + \hat{Q}_{V_\alpha} + (1 - \gamma_{\alpha,p}) \sum_{l=1}^{N^{p+}_1} (A_{l,N^p_l}\beta_{l,N^p_l} + F_{l,N^p_l}) \Big\rangle; \end{aligned} \tag{30}$$

$$\begin{aligned} D_\alpha = \frac{1}{h_t} V_\alpha \hat{\rho}_\alpha \hat{C}_{p_\alpha} - \sum_{j=1}^{N^+_\alpha} \sum_{k=1}^{M_{\beta\alpha}} \hat{\lambda}_{j,N_j}(E^k_{\beta\alpha})\, S_{j,N_j}(E^k_{\beta\alpha})\, [A_{j,N_j}(E^k_{\beta\alpha})\, \bar{\alpha}_{j,N_j}(E^k_{\beta\alpha}) - \\ - C_{j,N_j}(E^k_{\beta\alpha})] - \sum_{j=1}^{N^-_\alpha} \sum_{k=1}^{M_{\alpha\beta}} \lambda_{j,0}(E^k_{\alpha\beta})\, S_{j,0}(E^k_{\alpha\beta})\, [-C_{j,0}(E^k_{\alpha\beta}) + \\ + B_{j,0}(E^k_{\alpha\beta})\, \alpha_{j,0}(E^k_{\alpha\beta})] - (1 - \gamma_{\alpha,p}) \sum_{l=1}^{N^{p+}_1} \hat{\lambda}_{l,N^p_l} S_{l,N^p_l} (A_{l,N^p_l}\alpha_{l,N^p_l} - C_{l,N^p_l}). \end{aligned} \tag{31}$$

From the solution of system (28)-(31) we determine the unknown functions of temperature at the apices of the root graph, with consideration given to the thermal influence of the boundary graphs. Knowing T_α, $\forall\alpha \in V_r$, and utilizing (26) and (27), we will calculate the temperature distribution on all of the arms of the root graph.

Having found $T \in D_r$, we subsequently examine each p-th boundary graph. The temperature at the apices of the root graph $\alpha \in (V_r \cap V_p)$, $p = \overline{1, N_b}$, with which the p-th boundary graph is in contact, is used to calculate the temperature fields by means of (12) on the root arms of each boundary graph. As a result, we determine the temperatures of the first-level apices of the boundary graph, since $T_{j,N_j} = T_\alpha$, where $\alpha = j$. Using these values as the boundary conditions of the first kind, let us calculate the temperature fields at the second-level arms. Thus, as we make the sequential transition from the lower level to the upper level, we will calculate the temperature fields on each boundary graph.

This concludes the calculation of the temperature fields in the structure under consideration, with the corresponding time interval. Let us note that the utilization of non-iteration solution conjugacy at the apices to calculate the temperature fields on the boundary graphs significantly reduces the number of operations and reduces the expenditure of computer time.

In conclusion, let us note that the universal solution algorithm considered here for problems of mathematical modeling of the thermal regime of a structure in one-dimensional approximation integrates both the possibilities and unique features of the solution algorithms for these problems, determined on general-form graphs and on graphs in the shape of a tree. The utilization of this algorithm is most expedient when the thermal model of the thermophysical system under consideration is formalized by a complex branched graph of greater dimensionality.

NOTATION

T, temperature; t, time, index of time; x, spatial coordinate, index of spatial coordinate; λ, thermal conductivity; C_p, C_m, heat capacity; ρ, density; S, area of the lateral cross section of the element; q_V, Q_V, functions of the source in the heat-conduction equation for a distributed element and in the heat-balance equations for a concentrated element, respectively; h, interval of the difference approximation along the corresponding coordinate; A, B, C, F, system coefficients of difference algebraic equations; D, region of problem determination; Γ, boundary of region D; V, set of apices of the graph, volume of the concentrated element; $G_\alpha = G_\alpha^+ + G_\alpha^-$, set of arms, conjugate with the apex α; N, number of elements, arms, apices, levels; T, values of the function T at the instant of time k; $\hat{T}_1$, value of the function T at the instant of time k + 1. Subscripts: α, β, V, apices; j, ℓ, arms; i, coordinate point at the j-th arm; b, p, boundary graphs; γ, level of tree-shaped graph; in, inside apex; b, boundary apex; r, root graph; k, element of the set connecting two fixed apices.

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